Guide to converting numerical equations into quantity equations for KCDB applications

When the KCDB was launched in the year 2000, the numerical equation format was chosen.

The KCDB was revised and extended in 2019 to include an interactive web platform. Several Consultative Committees requested the implementation of the quantity equation format in the revised KCDB, a format which is frequently used and required by accreditation bodies. This request was approved by CIPM members at the Consultative Committee President's meeting, which was held at the BIPM in June 2018.

This document describes the format of quantity equations. It also gives some examples of how to transform numerical equations into quantity equations. It is based on guidance previously issued by the Consultative Committee for Length and has been adapted to fit a more general purpose.

Examples below are given for two terms to illustrate the general concept. Other examples including higher order and more complex relations follow the same general principles and are provided at the end of the document.

Numerical equation

CMCs are currently registered in the KCDB as numerical equations, such as:

$$U = \mathrm{Q}[49, 0.083L] \text{ nm, with } L \text{ in millimetres,}$$

which is a shorthand format for

$$U = \sqrt{49^2 + 0.083L^2} \text{ nm, with } L \text{ in millimetres.}$$

Quantity equation

With the quantity equation, the mix of nano- and milli- prefixes and the requirement to lock them becomes redundant. The quantity equation for the same uncertainty, expressed in the shorthand format becomes

$$U = \mathrm{Q}[49 \text{ nm}, 83 \times 10^{-9}\, L]$$

which explicitly means

$$U = \sqrt{(49 \text{ nm})^2 + (83 \times 10^{-9} L)^2}\ .$$

As the factor 83×10^{-9} represents a relative uncertainty, the same equation may easily be applied to other orders of magnitude. For example, if L is given in mm, the factor (49 nm) would become (49×10^{-6} mm), and the uncertainty would consequently be expressed in mm.

Converting a numerical equation into quantity equation format

The principle of the conversion from numerical to quantity equation format is given below as an example for length, L.

Let

$$U = \sqrt{A^2 + (BL)^2}\ \mathrm{u}, \text{with } L \text{ in } \mathrm{v} \tag{1}$$

be a numerical equation, where A, B, L are dimensionless numbers and u and v are units of length.

Let

$$U = \sqrt{a^2 + (bl)^2} \tag{2}$$

be the desired corresponding quantity equation, where l is the quantity length whose value is expressed in L v.

The conversion is achieved by the two equations below:

$$a = A\,\mathrm{u} \tag{3}$$

$$b = B\,\frac{\mathrm{u}}{\mathrm{v}} \tag{4}$$

as is easily shown by substitution:

$$\sqrt{a^2 + (bl)^2} = \sqrt{(A\,\mathrm{u})^2 + \left(B\,\frac{\mathrm{u}}{\mathrm{v}}\,l\right)^2} = \sqrt{A^2 + \left(B\,\frac{l}{\mathrm{v}}\right)^2}\ \mathrm{u} = \sqrt{A^2 + (BL)^2}\ \mathrm{u}, \text{with } L \text{ in } \mathrm{v}$$

Example of conversion

Given the numerical equation

$$U = \sqrt{49^2 + (0.083L\,)^2}\ \text{nm with } L \text{ in millimetres}$$

then

$$A = 49, \quad B = 0.083, \quad \mathrm{u} = \mathrm{nm}, \quad \mathrm{v} = \mathrm{mm}$$

Therefore

$$a = A\,\mathrm{u} = 49\ \mathrm{nm}, \quad b = B\,\frac{\mathrm{u}}{\mathrm{v}} = 0.083\,\frac{\mathrm{nm}}{\mathrm{mm}} = 0.083 \times 10^{-6} = 83 \times 10^{-9}$$

resulting in the quantity equation

$$U = \sqrt{a^2 + (bl\,)^2} = \sqrt{(49\ \mathrm{nm})^2 + (83 \times 10^{-9} l\,)^2}$$

The following table gives some examples of equations presented in the former format and how the information should appear in the KCDB[1].

[1] In the examples given above for quantity equations the length *l* is expressed using a lowercase italic serif font, in alignment with ISO 80000-1:2009 Quantities and units — Part 1: General. For the KCDB, however, the open sans serif font is used in general. To avoid mixing *l* with l, it is recommended to express the length *L* in the table using an uppercase italic sans serif font.

Initial format		
N	R	S
U unit	Uncertainty Equation	Comment Uncertainty Equation
mK	0.001 *T*	*T* in K, uncertainty in mK
nm	(0.002 + 3E-05 *p*)	*p* in nm, uncertainty in nm
nm	(25 + 5E-03 *h*)	*h* in µm, uncertainty in nm
µm	Q[40, 3 *L*]	*L* in m, uncertainty in µm
nm	Q[22, 0.66 *L*]	*L* in mm, uncertainty in nm
nm	Q[70, 0.06 *L*, 14 *P*]	*L* in mm, *P* in µm, uncertainty in nm
nm	Q[2.4E-6/L, 2.2E-9, 4.6E-4*α*]	*L* in mm, *α* in 1/K, uncertainty in nm
s/s	0.228/*t*	
%	(*Qm* + 0.05)	*Qm* in g/min, uncertainty in %
%	(0.005/*Qm* + 0.06)	*Qm* in g/min, uncertainty in %
%	0.10/(1-exp(-*q*/3.4))	*q* in mL/h, uncertainty in %
%rh	0.0035 × RH-reading + 0.15 %rh	

Targeted format				
L	M	N	R	S
U min	*U* max	*U* unit	Uncertainty Equation	Comment Uncertainty Equation
tbc	*tbc*	mK	1×10^{-6} *T*	*T* representing temperature
tbc	*tbc*	nm	0.002 nm + 3E-05 *p*	*p* representing pitch
tbc	*tbc*	nm	25 nm + 5E-06 *h*	*h* representing grating interval
tbc	*tbc*	µm	Q[40 µm, 3 E-06 *L*]	*L* representing pitch
tbc	*tbc*	nm	Q[22 nm, 6.6E-7 *L*]	*L* representing pitch
tbc	*tbc*	nm	Q[70 nm, 6E-8 *L*, 1.4E-2 *P*]	*L* representing pitch, *P* representing parallelism
tbc	*tbc*	nm	Q[2.2E-9 nm, 2.4 nm^2/ *L*, 4.6E-4 *α* K nm]	*L* representing length, *α* representing thermal expansivity
tbc	*tbc*	s/s	(0.228 s)/ *t*	*t* representing time
tbc	*tbc*	%	0.05 % + *Qm* /(g/min)	*Qm* representing flowrate in g/min
tbc	*tbc*	%	0.06 % + (0.005 g/min) / *Qm*	*Qm* representing flowrate in g/min
tbc	*tbc*	%	0.1 % /[1 - exp(-*q* / (3.4 mL/h))]	*q* representing flowrate
tbc	*tbc*	%rh	0.15 %rh + 3.5E-3 × *RH-reading*	*RH* representing relative humidity